

Sec 1

sheet 1

Math 4

PAGE

DATE

III $\phi = xy^2 + yz^3$ at point $(2, -1, 1)$

parallel vector $\vec{i} + 2\vec{j} + 2\vec{k}$

Sol.

$$D_{\vec{a}} \phi(x, y, z) = \nabla \phi|_p \cdot \frac{\vec{a}}{\|\vec{a}\|} = (1, -3, 3) \cdot \frac{(1, 2, 2)}{3} = \frac{-11}{3}$$

$$\nabla \phi = y^2 \vec{i} + (2xy + z^3) \vec{j} + (3yz^2) \vec{k}$$

$$\nabla \phi|_{(2, -1, 1)} = \vec{i} - 3\vec{j} - 3\vec{k} = (1, -3, 3)$$

IV) find $\nabla \phi$ $\phi = xyz$ $\phi = (x^2 + y^2 + z^2)^{-1/2}$

$$\nabla \phi = yz \vec{i} + xz \vec{j} + xy \vec{k}$$

$$\nabla \phi = \frac{1}{2} (2x)^{-3/2} \vec{i} + \frac{1}{2} (2y)^{-3/2} \vec{j} + \frac{1}{2} (2z)^{-3/2} \vec{k}$$

$$\nabla \phi = -\frac{1}{2} (x^2 + y^2 + z^2)^{-3/2} \cdot 2x = -\frac{1}{2} (x^2 + y^2 + z^2)^{-3/2} \cdot 2y$$

$$-\frac{1}{2} (x^2 + y^2 + z^2)^{-3/2} \cdot 2z$$

$$= -x (x^2 + y^2 + z^2)^{-3/2} - y (x^2 + y^2 + z^2)^{-3/2} - z (x^2 + y^2 + z^2)^{-3/2}$$

11

$$\phi = x^2 + y^2 + z^2$$

Point (1, 2, 3)

straight line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

$$\vec{\nabla}\phi = 2x\vec{i} + 2y\vec{j} + 2z\vec{k} \quad \left\{ \begin{array}{l} \text{From parametric eq.} \\ \text{at point (1, 2, 3)} \end{array} \right.$$

at point (1, 2, 3)

$$\vec{\nabla}\phi = 2\vec{i} + 4\vec{j} + 6\vec{k}$$

$$x = x_0 + a_x t$$

$$y = y_0 + a_y t$$

$$z = z_0 + a_z t$$

Symmetric eq.

$$\frac{\vec{\nabla}\phi}{a} = (2, 4, 6) \left(\frac{3, 4, 5}{5\sqrt{2}} \right) \therefore \frac{x - x_0}{a_x} = \frac{y - y_0}{a_y} = \frac{z - z_0}{a_z}$$

$$= \frac{52}{5\sqrt{2}}$$

$$\therefore \vec{a} = (3, 4, 5)$$

$$\|\vec{a}\| = \sqrt{9 + 16 + 25} = 5\sqrt{2}$$

[9] The unit vector normal to the surface $x^2 y^2 z^3 = 8$ at the point (1, 1, 2)

$$\phi = x^2 y^2 z^3 - 8$$

$$\vec{\nabla}\phi = 2xy^2z^3\vec{i} + 2yx^2z^3\vec{j} + 3z^2x^2y^2\vec{k}$$

at (1, 1, 2)

$$= 16\vec{i} + 16\vec{j} + 12\vec{k}$$

$$\hat{n} = \frac{\vec{\nabla}\phi}{\|\vec{\nabla}\phi\|}$$

$$\hat{n} = \frac{16\vec{i}}{4\sqrt{41}} + \frac{16\vec{j}}{4\sqrt{41}} + \frac{12\vec{k}}{4\sqrt{41}}$$

$$\|\vec{\nabla}\phi\| = \sqrt{16^2 + 16^2 + 12^2} = 4\sqrt{41}$$

$$= \frac{4}{\sqrt{41}}\vec{i} + \frac{4}{\sqrt{41}}\vec{j} + \frac{3}{\sqrt{41}}\vec{k}$$

$$(8) \quad u = x + y + z \quad v = x + y \quad w = -2xz - z^2 - 2yz$$

prove The triple scalar product

$$[\vec{\nabla} u, \vec{\nabla} v, \vec{\nabla} w] = 0$$

$$\vec{\nabla} u = \vec{i} + \vec{j} + \vec{k} \quad \vec{\nabla} v = \vec{i} + \vec{j}$$

$$\vec{\nabla} w = -2z\vec{j} - 2x\vec{k} - 2z\vec{k}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -2z & -2x & -2z \end{vmatrix} = \cancel{(-2z)} - \cancel{(-2z)} - \cancel{(-2z)}$$

$$-2y - 2x - 2z$$

$$= (-2y - 2x - 2z) - (-2y - 2x - 2z)$$

$$(-2z) - (-2z) = 0$$